

Structural Properties of Optimal Coordinate-Convex Policies for CAC with Nonlinearly-Constrained Feasibility Regions

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Abstract—Necessary optimality conditions for Call Admission Control (CAC) problems with nonlinearly-constrained feasibility regions and two classes of users are derived. The policies are restricted to the class of coordinate-convex policies. Two kinds of structural properties of the optimal policies and their robustness with respect to changes of the feasibility region are investigated: 1) general properties not depending on the revenue ratio associated with the two classes of users and 2) more specific properties depending on such a ratio. The results allow one to narrow the search for the optimal policies to a suitable subset of the set of coordinate-convex policies.

Index Terms—Call Admission Control, Feasibility Region, Coordinate Convex Policies.

I. INTRODUCTION

Call Admission Control (CAC) represents an important mechanism to guarantee specific Quality of Service (QoS) requirements. CAC determines when to accept or reject a new connection, flow, or call request (depending on which technology is used), thus limiting the load that enters a network. This is accomplished by verifying if enough resources are available to satisfy the performance requirements of a new call without penalizing the ones already in progress.

A basic model for CAC is the *stochastic knapsack* [1]. In this model, one has C units of resources and K classes of users. The calls from each class $k \in \{1, \dots, K\}$ arrive according to a Poisson process. If accepted by the system, each of them occupies b_k units of resources, which are released at the end of the call. The simplest CAC policy, known as *Complete Sharing (CS)*, consists in accepting a call whenever the system has sufficient resources. However CS may lead to a monopolistic use of resources by certain classes of users, thus to a poor resource utilization [2, Section III]. This motivates the interest in different admission policies (see the references in [3, Section 7.1]).

In general, finding optimal policies for the stochastic knapsack model is a difficult combinatorial optimization problem [4, Chapter 4]. The knowledge of structural properties of the optimal policies is useful to simplify its solution or at least to find good suboptimal policies. For instance, for two classes of users and an objective given by a weighted sum of per-class average revenues, structural properties were derived in [1] for the optimal policies belonging to the class of *coordinate-convex policies*. Coordinate-convex policies form a large class

of CAC policies characterized by a relatively simple structure and interesting properties, such as their product-form steady-state distribution [4, Chapter 4] and bounds on the per-class blocking probabilities [5]. When the service rates and resource requirements do not depend on the customer's class (*single service*), the optimal CAC policy is not coordinate-convex and is called *trunk reservation* ([6], [7]). For such a case, [8] and [9] provide recursive formulas to evaluate the performance of any trunk reservation policy and an iterative search algorithm to find optimal policies. They exploit such an algorithm to find coordinate optimal *threshold policies* (a particular kind of coordinate-convex policies) in *multiservice systems* (for which different classes may have different and heterogeneous resource requirements and mean service times).

The stochastic knapsack model can be extended by introducing the concept of *feasibility region* [10, pp. 46-49]. This is a region Ω_{FR} in the call space where given QoS requirements in terms of packet-loss/packet-delay probability are statistically guaranteed. When some form of *statistical multiplexing* is used, often this implies that the linear constraint $\{\sum_{k \in K} n_k b_k \leq C\}$ for the stochastic knapsack model is replaced by a more complicated constraint for the feasibility region, e.g. of the form $\sum_{k \in K} \beta_k(n_k) \leq C$ [4, p. 212], where the $\beta_k(\cdot)$ are nonlinear functions (see Figure 1).

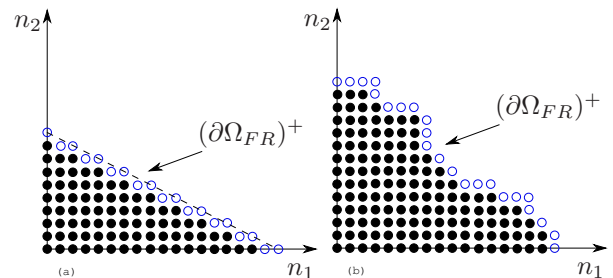


Figure 1. The upper boundary $(\partial\Omega_{FR})^+$ of a feasibility region Ω_{FR} with 2 class or users in the case of (a) a linearly-constrained Ω_{FR} (stochastic knapsack) and (b) a nonlinearly-constrained Ω_{FR} .

Up to our knowledge, till now the problem of finding structural properties of the optimal coordinate-convex policies in the case of general nonlinearly-constrained feasibility regions has received little attention, with the exception of [11], [12]

(see Subsection III-B for a comparison of our results with those of [12]).

The following is a summary of our contributions. For CAC problems with nonlinearly-constrained feasibility regions Ω_{FR} and two classes of users, we provide

- some general structural properties holding for any optimal coordinate-convex policy (Subsection III-A);
- more specific structural properties dependent on the revenue ratio associated with the two classes of users;
- simulation results (Section IV).

In doing so, we extend some results of [1] to nonlinearly-constrained feasibility regions; see Subsections III-A and III-B. All the proofs are deferred to Section VI.

II. PROBLEM FORMULATION

In this section, we summarize the CAC problem studied in [1], which will be extended in Section III to a nonlinearly-constrained feasibility region Ω_{FR} .

The state of the CAC system in [1] is described by a 2-dimensional vector \mathbf{n} , whose component n_k , $k = 1, 2$, represents the number of connections from users of class k that have been accepted by the system and are currently in progress. For each class k , the inter-arrival times are exponentially distributed with mean value $1/\lambda_k(n_k)$ and the holding times of accepted connections are independent and identically distributed (i.i.d.) with mean value $1/\mu_k$. The CAC system accepts or rejects a request of connection according to a coordinate-convex policy. Here we recall its definition [4, p. 116].

Definition II.1. A nonempty set $\Omega \subseteq \Omega_{FR} \subset \mathbb{N}_0^2$ is called *coordinate-convex* if and only if it has the following property: for each $\mathbf{n} \in \Omega$ with $n_k > 0$ one has $\mathbf{n} - \mathbf{e}_k \in \Omega$, where \mathbf{e}_k is a 2-dimensional vector whose k -th component is 1 and the other one is 0. The coordinate-convex policy associated with a coordinate-convex set Ω admits an arriving request of connection if and only if after admittance the state process remains in Ω .

As there is a one-to-one correspondence between coordinate-convex sets and coordinate-convex policies, from now on we use the symbol Ω to denote either a coordinate-convex set or a coordinate-convex policy.

The objective to be maximized by the CAC system in the space $\mathcal{P}(\Omega_{FR})$ of coordinate-convex subsets of Ω_{FR} is given by

$$J(\Omega) = \sum_{\mathbf{n} \in \Omega} (\mathbf{n} \cdot \mathbf{r}) P_{\Omega}(\mathbf{n}), \quad (1)$$

where \mathbf{r} is a 2-dimensional vector whose component r_k represents the instantaneous positive revenue generated by any accepted connection of class k that is still in progress and $P_{\Omega}(\mathbf{n})$ is the steady-state probability that the CAC system is in state \mathbf{n} . As Ω is coordinate-convex, $P_{\Omega}(\mathbf{n})$ takes on the product-form expression

$$P_{\Omega}(\mathbf{n}) = \frac{\prod_{i=1}^2 q_i(n_i)}{\sum_{\mathbf{n} \in \Omega} \prod_{i=1}^2 q_i(n_i)}, \quad (2)$$

where

$$q_i(n_i) := \frac{\prod_{j=0}^{n_i-1} \lambda_i(j)}{n_i! \mu_i^{n_i}}. \quad (3)$$

For linearly-constrained feasibility regions Ω_{FR} , [1] describes structural properties of the coordinate-convex policies maximizing the objective (1) (e.g., the existence of one vertical threshold, one horizontal threshold, or both kinds of thresholds), which depend on the value assumed by the revenue ratio $R := r_2/r_1$.

III. MAIN RESULTS

In our analysis, we allow the feasibility region Ω_{FR} to have a nonlinear upper boundary, denoted by $(\partial\Omega_{FR})^+$ (see Figure 1(b)). Similarly, we denote by $(\partial\Omega)^+$ the (linear or nonlinear) upper boundary of the coordinate-convex set Ω . The set Ω_{FR} is assumed to be coordinate-convex, as it often happens for feasibility regions defined in terms of QoS constraints [13, Proposition 6.3]. Let us recall two definitions from [1].

Definition III.1. The tuple $(\alpha, \beta) \in \Omega_{FR} \setminus \Omega$ is a *type-1 corner point* for Ω if and only if $\beta \geq 1$, $(\alpha, \beta - 1) \in \Omega$, and either $\alpha = 0$ or $(\alpha - 1, \beta) \in \Omega$; the tuple $(\alpha, \beta) \in \Omega_{FR} \setminus \Omega$ is a *type-2 corner point* for Ω if and only if $\alpha \geq 1$, $(\alpha - 1, \beta) \in \Omega$, and either $\beta = 0$ or $(\alpha, \beta - 1) \in \Omega$.

Definition III.2. A nonempty set $S^- \subset \Omega_{FR}$ is *incrementally removable* with respect to Ω (IR_{Ω}) if and only if $S^- \subset \Omega$ and $\Omega \setminus S^-$ is still a coordinate-convex set; a nonempty set $S^+ \subset \Omega_{FR}$ is *incrementally admissible* with respect to Ω (IA_{Ω}) if and only if $S^+ \cap \Omega = \emptyset$ and $\Omega \cup S^+$ is still a coordinate-convex set.

In the following, we shall sometimes use the term ‘‘corner point’’ to refer to either a type-1 or a type-2 corner point. By the coordinate-convexity of Ω , no two corner points can be on the same vertical or horizontal line.

We recall from [1] that the definition of the objective $J(\cdot)$ in (1) can be extended consistently to all (not necessarily coordinate-convex) sets $S \subseteq \Omega_{FR}$ in the following way:

$$J(S) := \frac{H(S)}{G(S)}, \quad (4)$$

with

$$H(S) := \sum_{\mathbf{n} \in S} (\mathbf{n} \cdot \mathbf{r}) \prod_{i=1}^2 q_i(n_i), \quad (5)$$

$$G(S) := \sum_{\mathbf{n} \in S} \prod_{i=1}^2 q_i(n_i). \quad (6)$$

We also recall that, for a rectangular region $S := \{a, a + 1, \dots, b\} \times \{c, c + 1, \dots, d\}$, by (3), (5), and (6) it follows

$$J(S) = r_1 x_1(a, b) + r_2 x_2(c, d), \quad (7)$$

where

$$x_i(a, b) := \frac{\sum_{k=a}^b k q_i(k)}{\sum_{j=a}^b q_i(j)}. \quad (8)$$

A. General structural properties of the optimal coordinate-convex policies

Let Ω° denote any optimal coordinate-convex policy (or its associated coordinate-convex set). Proposition III.3, which extends to nonlinearly-constrained feasibility regions a similar property stated in [1, Theorem 1] for linearly-constrained ones, states that the corner points of Ω° are to be searched among the vertices of a suitable grid (see Figure 2). We use the following notations:

$$l_2^\Omega(n_1) := \max\{k \in \mathbb{N}_0 \text{ such that } (n_1, k) \in \Omega\}, \quad (9)$$

$$l_1^\Omega(n_2) := \max\{h \in \mathbb{N}_0 \text{ such that } (h, n_2) \in \Omega\}. \quad (10)$$

The values $l_1^\Omega(n_2)$ and $l_2^\Omega(n_1)$ are, respectively, the maximum number of type-1/type-2 connections allowed in Ω when we have already n_2 type-2/ n_1 type-1 connections. It follows from the definitions that the functions $l_i^\Omega(\cdot)$ are nonincreasing. Set $n_{1,\max} := l_1^{\Omega_{FR}}(0)$, $n_{2,\max} := l_2^{\Omega_{FR}}(0)$, $l_1^{\Omega_{FR}}(n_{2,\max} + 1) + 1 := 0$ and $l_2^{\Omega_{FR}}(n_{1,\max} + 1) + 1 := 0$.

Proposition III.3. (i) If (α, β) is a type-2 corner point for Ω° , then for some $j = 1, \dots, n_{2,\max}$ we have

$$\alpha = l_1^{\Omega_{FR}}(j) + 1. \quad (11)$$

(ii) If (α, β) is a type-1 corner point for Ω° , then for some $j = 1, \dots, n_{1,\max}$ we get

$$\beta = l_2^{\Omega_{FR}}(j) + 1. \quad (12)$$

We refer to the reader to [14] for other general structural properties of the optimal coordinate-convex policies, not depending on the revenue ratio R .

B. Structural properties of the optimal coordinate-convex policies depending on the revenue ratio R

Let us now consider for the optimal coordinate-convex policies structural properties obtained for suitable values of the revenue ratio R . We recall the following definition from [1].

Definition III.4. Let $i = 1$ or 2 . A coordinate-convex policy Ω is threshold type- i if and only if for some $t_i = 0, \dots, n_{i,\max}$ we get

$$\Omega = \{(n_1, n_2) \in \Omega_{FR} : n_i \leq t_i\}. \quad (13)$$

Proposition III.5 is our extension of [1, Lemma 4] to general nonlinearly-constrained feasibility regions. With respect to [1], due to the different shape of the feasibility region, in general it is not true that $j \neq k$ implies $l_1^{\Omega_{FR}}(j) \neq l_1^{\Omega_{FR}}(k)$. As shown in Figure 2, in general for every $j_2 \in \{0, \dots, n_{2,\max}\}$ there exist $j^{(2,l)} \leq j_2$ and $j^{(2,u)} \geq j_2$ such that $l_1^{\Omega_{FR}}(\cdot)$ is constant on the set $\{j^{(2,l)}, \dots, j^{(2,u)}\} \subseteq \{0, \dots, n_{2,\max}\}$. Similarly, for every $j_1 \in \{0, \dots, n_{1,\max}\}$ there exist $j^{(1,l)} \leq j_1$ and $j^{(1,u)} \geq j_1$ such that $l_2^{\Omega_{FR}}(\cdot)$ is constant on the set $\{j^{(1,l)}, \dots, j^{(1,u)}\} \subseteq \{0, \dots, n_{1,\max}\}$. Let $B_1 := \max\{j^{(1,u)} - j^{(1,l)} : j_1 = 0, \dots, n_{1,\max}\}$ and $B_2 := \max\{j^{(2,u)} - j^{(2,l)} : j_2 = 0, \dots, n_{2,\max}\}$. Recall that $R := r_2/r_1$ is the revenue ratio.

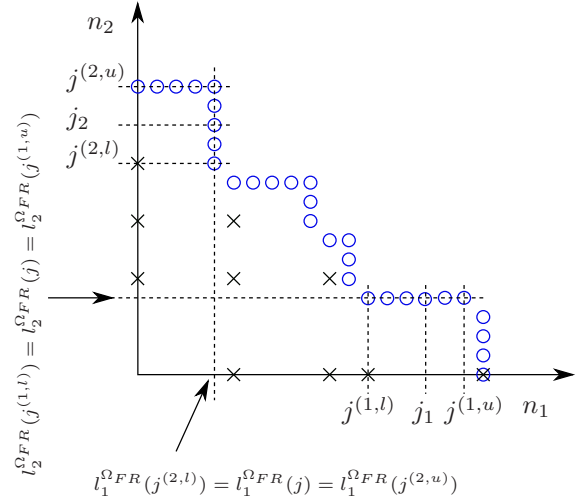


Figure 2. Potential locations of the corner points of an optimal coordinate-convex policy Ω° . According to Proposition III.3, the corner points of Ω° are to be searched among the crosses in the figure.

Proposition III.5. (i) If (α, β) is a type-2 corner point for Ω° and $\lambda_2(\cdot)$ is nonincreasing, then for some $j = 1, \dots, n_{2,\max}$ (11) holds together with

$$Rx_2(0, B_2) \geq x_1(l_1^{\Omega_{FR}}(j^{(2,l)}) + 1, l_1^{\Omega_{FR}}(j^{(2,l)} - 1)) - x_1(l_1^{\Omega_{FR}}(j^{(2,u)} + 1) + 1, l_1^{\Omega_{FR}}(j^{(2,u)})). \quad (14)$$

(ii) If (α, β) is a type-1 corner point for Ω° and $\lambda_1(\cdot)$ is nonincreasing, then for some $j = 1, \dots, n_{1,\max}$ (12) holds together with

$$\frac{1}{R}x_1(0, B_1) \geq x_2(l_2^{\Omega_{FR}}(j^{(1,l)}) + 1, l_2^{\Omega_{FR}}(j^{(1,l)} - 1)) - x_2(l_2^{\Omega_{FR}}(j^{(1,u)} + 1) + 1, l_2^{\Omega_{FR}}(j^{(1,u)})). \quad (15)$$

The following theorem states that under suitable conditions one has threshold-type optimal policies. The result is an extension of [1, Theorem 1] to feasibility regions with a nonlinear upper boundary. Its proof exploits Proposition III.5.

Theorem III.6. Let $\lambda_i(\cdot)$ be nonincreasing for $i = 1, 2$.

(i) If $\frac{1}{R} < L_1$, where

$$L_1 := \min_{j=1, \dots, n_{1,\max}} \left\{ \frac{x_2(l_2^{\Omega_{FR}}(j^{(1,l)}) + 1, l_2^{\Omega_{FR}}(j^{(1,l)} - 1))}{x_1(0, B_1)} - \frac{x_2(l_2^{\Omega_{FR}}(j^{(1,u)} + 1) + 1, l_2^{\Omega_{FR}}(j^{(1,u)}))}{x_1(0, B_1)} \right\}, \quad (16)$$

then Ω° is threshold type-1, and the threshold is equal to some $l_1^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{2,\max}$.

(ii) If $R < L_2$, where

$$L_2 := \min_{j=1, \dots, n_{2, \max}} \left\{ \frac{x_1(l_1^{\Omega_{FR}}(j^{(2,l)} + 1) + 1, l_1^{\Omega_{FR}}(j^{(2,l)} - 1))}{x_2(0, B_2)} - \frac{x_1(l_1^{\Omega_{FR}}(j^{(2,u)} + 1) + 1, l_1^{\Omega_{FR}}(j^{(2,u)}))}{x_2(0, B_2)} \right\}, \quad (17)$$

then Ω° is threshold type-2, and the threshold is equal to some $l_2^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{1, \max}$.

(iii) If $\frac{1}{L_1} < R < L_2$, then $\Omega^\circ = \Omega_{FR}$.

The following corollary provides sufficient conditions for threshold-type optimal policies.

Corollary III.7. Let $\lambda_i(\cdot)$ be nonincreasing for $i = 1, 2$.

(i) If $R > x_1(0, B_1)$, then Ω° is threshold type-1, and the threshold is equal to some $l_1^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{2, \max}$.

(ii) If $\frac{1}{R} > x_2(0, B_2)$, then Ω° is threshold type-2, and the threshold is equal to some $l_2^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{1, \max}$.

(iii) If $x_1(0, B_1) < R < \frac{1}{x_2(0, B_2)}$, then $\Omega^\circ = \Omega_{FR}$.

Remark III.8. In the particular case of a linearly-constrained feasibility region with $B_2 = 1$ (i.e., the one considered in [1]), one has $j^{(1,l)} = j^{(1,u)}$ for each $j = 0, \dots, n_{1, \max}$, and $L_1 = \frac{1}{x_1(0, B_1)}$. So in this case Theorem III.6 (i) reduces to [1, Theorem 1] (i).

Remark III.9. Another extension of [1, Theorem 1] which is similar to Proposition III.3 and Corollary III.7 is reported in [11, Section 4] and refers to a less general nonlinearly-constrained feasibility region and to a different assumption on the holding time distribution of the calls.

IV. SIMULATION RESULTS

In the next numerical results we show that, under the conditions of Corollary III.7 (i) and (ii) resp., the optimal threshold for threshold type-1 policies is indeed equal to $l_1^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{2, \max}$, and the optimal threshold for threshold type-2 policies is equal to $l_2^{\Omega_{FR}}(j)$ for some $j = 0, \dots, n_{1, \max}$. In Figure 3 the feasibility region used to make these simulations is depicted. We assume homogeneous Poisson arrivals for both classes.

With this feasibility region we have $B_1 = 16$ and $B_2 = 10$; for $\lambda_1 = 50$, $\lambda_2 = 150$, $\mu_1 = 0.5$, $\mu_2 = 5$, $r_1 = 0.25$, and $r_2 = 2.5$ we have $R = r_2/r_1 = 10$ and $x_1(0, 10) \simeq 9.89$. Then $R > x_1(0, B_1)$ and by Corollary III.7 (i) there exists an optimal coordinate-convex policy that is threshold type-1. According to Corollary III.7 (i), the optimal threshold belongs to the set $\{0, 15, 22, 28\}$. Figure 3 (b) shows that this is indeed the case, and that the optimal threshold is $t_1 = 15$.

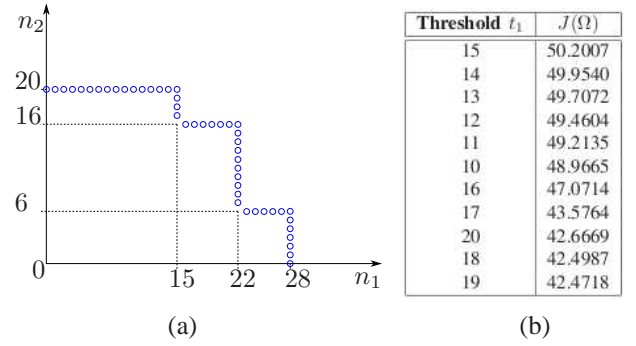


Figure 3. (a) The feasibility region considered in Section IV. (b)

V. CONCLUSIONS

This paper provides several structural properties of the corner points of the optimal coordinate-convex policies in CAC problems with nonlinearly-constrained feasibility regions and two classes of users. These properties can be used to narrow the search for the optimal coordinate-convex policies. For certain feasibility regions characterized by a nonlinear constraint of the form $\sum_{k \in K} \beta_k(n_k) \leq C$, the simplest possible extension of the results to more than 2 classes of users consists in defining subproblems obtained by partitioning the set of classes by using subsets of cardinality at most 2 and applying to each subproblem the results obtained here for 2 classes of users.

VI. PROOFS

Due to the page limits, we refer the reader to [1] for the statements of [1, Lemmas 1-3], which are used in some of the following proofs. An inspection of the proofs of [1, Lemmas 1-3] shows that they hold for both linearly-constrained and nonlinearly-constrained feasibility regions.

Proof of Proposition III.3. We prove only (i), since (ii) can be obtained in the same way by exchanging the roles of the two classes of users. Suppose that (11) is violated for every $j = 1, \dots, n_{2, \max}$. Choosing $n = l_2^{\Omega^\circ}(\alpha - 1) - \beta \geq 0$, $S^-(n) = \{(\alpha - 1, \beta + i) : i = 0, \dots, n\} \subseteq \Omega^\circ$, and $S^+(n) = \{(\alpha, \beta + i) : i = 0, \dots, n\} \subseteq \Omega_{FR} \setminus \Omega^\circ$ (see Figure 4), it follows that the sets $\Omega^\circ \setminus S^-(n)$ and $\Omega^\circ \cup S^+(n)$ are coordinate-convex, so $S^-(n)$ is IR_{Ω° and $S^+(n)$ is IA_{Ω° . By formula (7), we get $J(S^-(n)) = r_1(\alpha - 1) + r_2 x_2(\beta, \beta + n) < r_1 \alpha + r_2 x_2(\beta, \beta + n) = J(S^+(n))$, but this contradicts the optimality condition stated in [1, Lemma 2], so one concludes that there exists some $j = 1, \dots, n_{2, \max}$ such that (11) holds. ■

Proof of Proposition III.5. Given a type-2 corner point (α, β) , we know from Proposition III.3 (i) that $\alpha = l_1^{\Omega_{FR}}(j) + 1$ for some $j = 1, \dots, n_{2, \max}$. Choosing $n = l_2^{\Omega^\circ}(\alpha - 1) - \beta \geq 0$, $m = \min\{l_2^{\Omega_{FR}}(\alpha - \beta), n\}$, $\hat{S}^-(n) = \{l_1^{\Omega_{FR}}(j^{(2,u)} + 1) + 1, \dots, l_1^{\Omega_{FR}}(j^{(2,u)})\} \times \{\beta, \dots, \beta + n\} \subseteq \Omega^\circ$, and $\hat{S}^+(m) = \{l_1^{\Omega_{FR}}(j^{(2,l)} + 1, \dots, l_1^{\Omega_{FR}}(j^{(2,l)} - 1)\} \times \{\beta, \dots, \beta + m\} \subseteq \Omega_{FR} \setminus \Omega^\circ$ (see Figure 5), it follows that the sets $\Omega^\circ \setminus \hat{S}^-(n)$

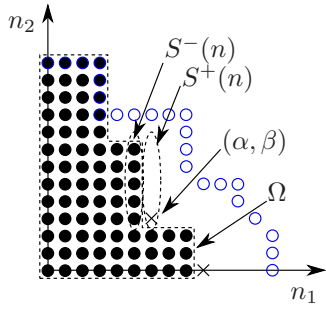


Figure 4. An example of a coordinate-convex set Ω having a type-2 corner point (α, β) for which $\alpha \neq l_1^{\Omega_{FR}}(j) + 1$ for every $j = 1, \dots, n_{2,\max}$.

and $\Omega^\circ \cup \hat{S}^+(m)$ are coordinate-convex, so $\hat{S}^-(n)$ is IR_{Ω° and $\hat{S}^+(m)$ is IA_{Ω° . By (7) we get

$$\begin{aligned} & J(\hat{S}^-(n)) \\ &= r_1 x_1 (l_1^{\Omega_{FR}}(j^{(2,u)} + 1) + 1, l_1^{\Omega_{FR}}(j^{(2,u)})) + r_2 x_2(\beta, \beta + n) \end{aligned}$$

and

$$\begin{aligned} & J(\hat{S}^+(m)) \\ &= r_1 x_1 (l_1^{\Omega_{FR}}(j^{(2,l)} + 1, l_1^{\Omega_{FR}}(j^{(2,l)} - 1)) + r_2 x_2(\beta, \beta + m). \end{aligned}$$

Combining these equalities with [1, Lemma 2] (which implies $J(\hat{S}^-(n)) \geq J(\hat{S}^+(n))$), we have

$$\begin{aligned} & R(x_2(\beta, \beta + n) - x_2(\beta, \beta + m)) \\ & \geq x_1 (l_1^{\Omega_{FR}}(j^{(2,l)} + 1, l_1^{\Omega_{FR}}(j^{(2,l)} - 1)) \\ & \quad - x_1 (l_1^{\Omega_{FR}}(j^{(2,u)} + 1, l_1^{\Omega_{FR}}(j^{(2,u)}))). \end{aligned} \quad (18)$$

Since $0 \leq n - m \leq B_2$ and $\lambda_2(\cdot)$ is nonincreasing, by [1, Lemma 3] we get

$$x_2(0, B_2) \geq x_2(\beta, \beta + n) - x_2(\beta, \beta + m)$$

which, when combined with (18), proves (14). Formula (ii) is obtained in the same way by exchanging the roles of the two classes of users. ■

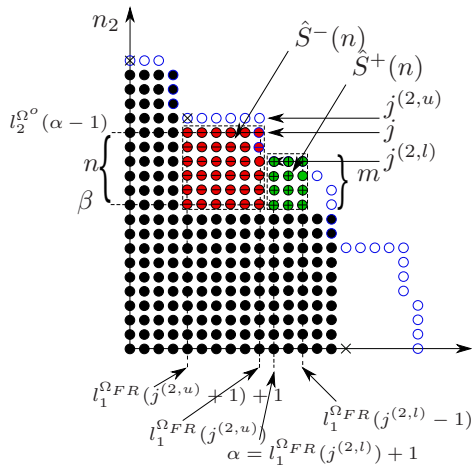


Figure 5. A description of a step in the proof of Proposition III.5.

Proof of Theorem III.6. (i) If $\frac{1}{R} < L_1$, then by Proposition III.5 (ii) Ω° has no type-1 corner points, so it is a threshold type-1 policy by [1, Lemma 1]. Let t_1 denote the corresponding threshold. Then either $t_1 = n_{1,\max} = l_1^{\Omega_{FR}}(0)$ or $(t_1 + 1, 0)$ is a type-2 corner point for Ω° . In the second case, by Proposition III.3 (i) we have $t_1 + 1 = l_1^{\Omega_{FR}}(j) + 1$ for some $j = 1, \dots, n_{2,\max}$.

(ii) is proved similarly.

(iii) If $\frac{1}{L_1} < R < L_2$, then by parts (i) and (ii) Ω° is both threshold type-1 and threshold type-2, so it coincides with Ω_{FR} . ■

Proof of Corollary III.7. For each $j = 0, \dots, n_{1,\max}$, it follows from the definitions of $x_2(\cdot, \cdot)$ and of $j^{(1,l)}, j^{(1,u)}$ that

$$\begin{aligned} x_2(l_2^{\Omega_{FR}}(j^{(1,l)} + 1, l_2^{\Omega_{FR}}(j^{(1,l)} - 1)) & \geq l_2^{\Omega_{FR}}(j^{(1,l)} + 1, \\ x_2(l_2^{\Omega_{FR}}(j^{(1,u)} + 1, l_2^{\Omega_{FR}}(j^{(1,u)})) & \leq l_2^{\Omega_{FR}}(j^{(1,u)}, \end{aligned}$$

and $l_2^{\Omega_{FR}}(j^{(1,u)}) = l_2^{\Omega_{FR}}(j^{(1,l)})$, so $L_1 \geq \frac{1}{x_1(0, B_1)}$. Similarly, we have $L_2 \geq \frac{1}{x_2(0, B_2)}$. ■

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