On call admission control with nonlinearly constrained feasibility regions

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Problem formulation

- State of the CAC system: 2-dimensional vector $\mathbf{n}$.
  - n_k ($k = 1, 2$): number of connections from users of class k that have been accepted and are currently in progress.
- Inter-arrival times: exponentially distributed with mean values $1/\lambda_k(n_k)$.
- Holding times of accepted connections: independent and identically distributed with mean $1/\mu_k$.
- The CAC system accepts or rejects a request of connection according to a policy.

Coordinate-convex sets and policies

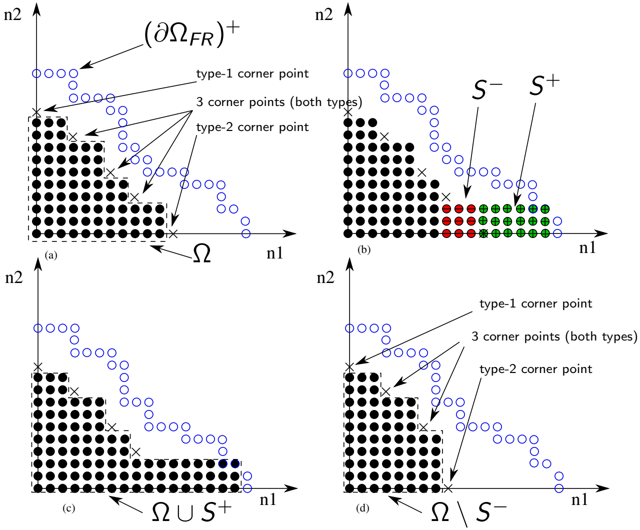
Definition 1

A nonempty set $\Omega \subset \mathbb{N}_0^2$ is called coordinate-convex (c.c.) iff it has the following property: for each $\mathbf{n} \in \Omega$ with $n_k > 0$ one has $\mathbf{n} - \mathbf{e}_k \in \Omega$, where $\mathbf{e}_k$ is a 2-dimensional vector whose k-th component is 1 and the other one is 0.

Definition 2

A c.c. policy with associated c.c. set Ω admits an arriving request of connection iff the state process remains in Ω after admittance.

Example


n2
$(\partial\Omega_{FR})^+$
type-1 corner point
3 corner points (both types)
type-2 corner point
(a)
Ω
n1
n2
S^-
S^+
(b)
n1
n2
(c)
$\Omega \cup S^+$
n1
n2
type-1 corner point
3 corner points (both types)
type-2 corner point
(d)
$\Omega \setminus S^-$
n1

Feasibility region

- $\Omega_{FR} \subset \mathbb{N}_0^2$ such that given Quality of Service (QoS) constraints are satisfied.
- Complete sharing policy: admit a new call if and only if the call state after its potential admittance is still within Ω_{FR}.
  - (Often) poor resource utilization.
- Consider other admission policies.

Optimization problem

Objective to be maximized:

$$J(\Omega) = \sum_{\mathbf{n} \in \Omega} (\mathbf{n} \cdot \mathbf{r}) P_{\Omega}(\mathbf{n})\,. \tag{1}$$

- $\mathbf{r}$: 2-dimensional vector whose component r_k represents the instantaneous revenue generated by any accepted connection of class k that is still in progress.
- $\Omega \subseteq \Omega_{FR}$ coordinate-convex.
- $P_{\Omega}(\mathbf{n})$: steady-state probability that the CAC system is in state $\mathbf{n}$ under the policy Ω.

Previous results for linearly-constrained feasibility regions

- (Ross and Tsang '89): structural properties of the c.c. policies maximizing the objective (1). Existence of
  - one (and only one) vertical threshold;
  - one (and only one) horizontal threshold;
  - both kinds of thresholds.
- Structural results dependent on the value assumed by the revenue ratio $R := r_2/r_1$.
- They may not hold anymore for nonlinearly-constrained Ω_{FR}.

Main objectives

- Give some characterization of the optimal policies in CAC problems with nonlinearly-constrained feasibility regions.
- Obtain theoretical results that can be applied to
  - narrow the search for the (unknown) optimal c.c. policies;
  - improve given suboptimal c.c. policies.

Incrementally removable sets, incrementally admissible sets, and corner points

Definition 3

A nonempty set $S^- \subset \Omega_{FR}$ is incrementally removable with respect to Ω (IR_Ω) iff $S^- \subset \Omega$ and $\Omega \setminus S^-$ is still a c.c. set.

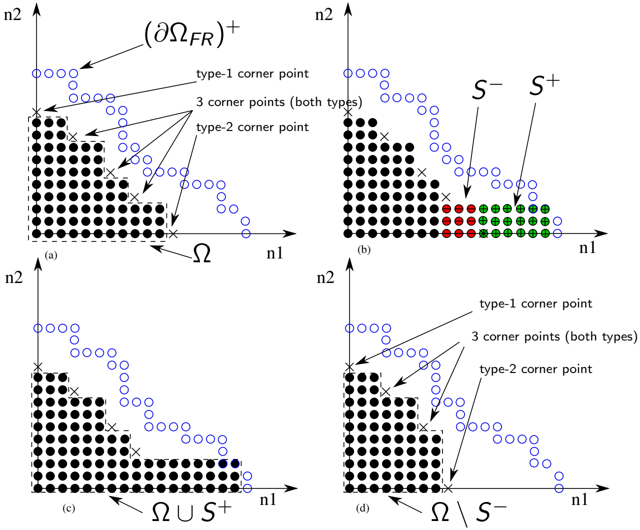
Definition 4

A nonempty set $S^+ \subset \Omega_{FR}$ is incrementally admissible with respect to Ω (IA_Ω) iff $S^+ \cap \Omega = \emptyset$ and $\Omega \cup S^+$ is still a c.c. set.

Definition 5

The tuple $(\alpha, \beta) \in \Omega_{FR} \setminus \Omega$ is a type-1 corner point for Ω iff $\beta \geq 1$, $(\alpha, \beta - 1) \in \Omega$, and either $\alpha = 0$ or $(\alpha - 1, \beta) \in \Omega$; the tuple $(\alpha, \beta) \in \Omega_{FR} \setminus \Omega$ is a type-2 corner point for Ω iff $\alpha \geq 1$, $(\alpha - 1, \beta) \in \Omega$, and either $\beta = 0$ or $(\alpha, \beta - 1) \in \Omega$.

Example



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Ω
(a)
S^-
S^+
(b)
$\Omega \cup S^+$
(c)
$\Omega \setminus S^-$
(d)

A criterion to improve suboptimal policies (I)

Proposition 1

Let (α, β) be a type-2 corner point for Ω and suppose that there exist $n, m, p \in \mathbb{N}_0$ such that
$S^- := \{(\alpha - 1 - j, \beta + i) : j = 0, \ldots, n,\ i = 0, \ldots, p\} \subset \Omega$, is IR_Ω, and
$S^+ := \{(\alpha + s, \beta + i) : s = 0, \ldots, m,\ i = 0, \ldots, p\} \subset \Omega_{FR}$, is IA_Ω. Then at least one of the following inequalities holds:
(i) $J(\Omega \cup S^+) > J(\Omega)$;
(ii) $J(\Omega \setminus S^-) > J(\Omega)$.

- Idea of the proof: extending an argument used in (Ross and Tsang '89, proof of Lemma 1).

A criterion to improve suboptimal policies (II)

Proposition 2

Let (α, β) be a type-1 corner point for Ω and suppose that there exist $n, m, p \in \mathbb{N}_0$ such that
$S^- := \{(\alpha + i, \beta - 1 - j) : i = 0, \ldots, p,\ j = 0, \ldots, n\} \subset \Omega$*, is* IA_Ω*, and*
$S^+ := \{(\alpha + i, \beta + s) : i = 0, \ldots, p,\ s = 0, \ldots, m\} \subset \Omega_{FR}$*, is* IA_Ω*. Then*
at least one of the following inequalities holds:
(i) $J(\Omega \setminus S^-) > J(\Omega)$*;*
(ii) $J(\Omega \cup S^+) > J(\Omega)$*.*

- Obtained by reversing the roles of the two classes of users.

A property of the corner points of any optimal policy

Proposition 3

The following holds.
(i) Let (α, β) *be a type-2 corner point of* Ω *for which Proposition 1 cannot be applied. Then* $l_2^{\Omega}(\alpha - 1) > l_2^{\Omega_{FR}}(\alpha)$.
(ii) Let (α, β) *be a type-1 corner point of* Ω *for which Proposition 2 cannot be applied. Then* $l_1^{\Omega}(\beta - 1) > l_1^{\Omega_{FR}}(\beta)$.

$$l_2^{\Omega}(n_1) := \max\{k \in \mathbb{N}_0 \text{ such that } (n_1, k) \in \Omega\}\,,$$
$$l_1^{\Omega}(n_2) := \max\{h \in \mathbb{N}_0 \text{ such that } (h, n_2) \in \Omega\}\,.$$

- Any policy Ω^* that cannot be further improved via Proposition 1 or 2 (in particular, any optimal policy) can have only the kind of corner points described in Proposition 3.

A structural property of any optimal policy (I)

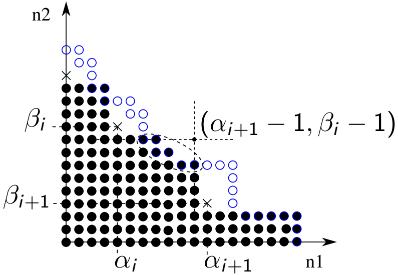
Proposition 4

Let (α_i, β_i) and $(\alpha_{i+1}, \beta_{i+1})$ two consecutive corner points (if present) of Ω^*. Then the intersection point $(\alpha_{i+1} - 1, \beta_i - 1)$ between the vertical line $n_1 = \alpha_{i+1} - 1$ and the horizontal line $n_2 = \beta_i - 1$ either lies on $(\partial\Omega_{FR})^+$, or is outside Ω_{FR}.

- Idea of the proof: the statement is equivalent to the pair of inequalities

$$l_1^{\Omega_{FR}}(\beta_i - 1) \leq \alpha_{i+1} - 1 \,,$$
$$l_2^{\Omega_{FR}}(\alpha_{i+1} - 1) \leq \beta_i - 1 \,.$$

A structural property of any optimal policy (II)

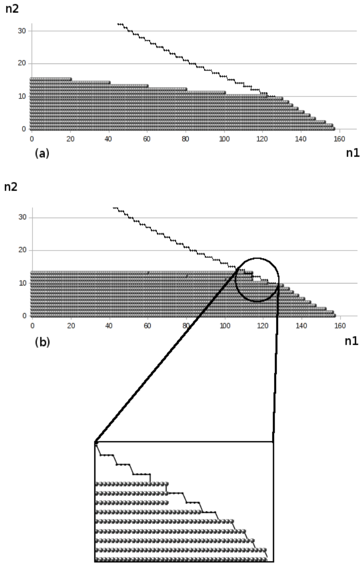


- Between any two successive corner points of Ω^* there is a nonempty intersection between $(\partial\Omega^*)^+$ and $(\partial\Omega_{FR})^+$ (dotted ellipse).

Simulation results (I)

- Poisson arrivals for both classes, $\lambda_1 = 6$, $\lambda_2 = 20$, $\mu_1 = \mu_2 = 0.1$, and $r_1 = r_2 = 1$.
- Starting from the initial Ω_1, the final policy Ω_6 is obtained by applying Proposition 3 5 times to suitable corner points. The initial value of the objective is $J(\Omega_1) = 66.4229$, whereas the final value is $J(\Omega_6) = 72.9313$, with an improvement of 9.8%.
- Note that the final policy Ω_6 cannot be further improved via Propositions 1 or 2, and that it has the structural property stated in Proposition 3.

Simulation results (II)



Conclusions

- Some characterization of the optimal policies in CAC problems with nonlinearly-constrained feasibility regions.
- The theoretical results can be applied to
  - narrow the search for the (unknown) optimal c.c. policies;
  - improve given suboptimal c.c. policies.
- Possible extension to $K > 2$ class of users: partition the set $\{1, \ldots, K\}$ by using subsets of cardinality at most 2.

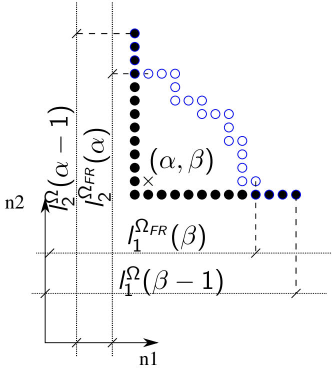
Thanks for the attention

Extra slides

A property of the corner points of any optimal policy (I)

- Maximum number of type-1/type-2 connections allowed in Ω when we have already n_2 type-2/n_1 type-1 connections.

$$l_2^{\Omega}(n_1) := \max\{k \in \mathbb{N}_0 \text{ such that } (n_1, k) \in \Omega\}\,,$$
$$l_1^{\Omega}(n_2) := \max\{h \in \mathbb{N}_0 \text{ such that } (h, n_2) \in \Omega\}\,.$$



A property of the corner points of any optimal policy (II)

Proposition 3

The following holds.
(i) Let (α, β) be a type-2 corner point of Ω for which Proposition 1 cannot be applied. Then $l_2^{\Omega}(\alpha - 1) > l_2^{\Omega_{FR}}(\alpha)$.
(ii) Let (α, β) be a type-1 corner point of Ω for which Proposition 2 cannot be applied. Then $l_1^{\Omega}(\beta - 1) > l_1^{\Omega_{FR}}(\beta)$.

- Any policy Ω^* that cannot be further improved via Proposition 1 or 2 (in particular, any optimal policy) can have only the kind of corner points described in Proposition 3.

Proof of Proposition 1

Suppose for now that neither $J(\Omega \cup S^+) > J(\Omega)$ nor $J(\Omega \setminus S^-) > J(\Omega)$ holds. Then,

$$J(\Omega \cup S^+) = \frac{H(\Omega) + H(S^+)}{G(\Omega) + G(S^+)} \leq J(\Omega) = \frac{H(\Omega)}{G(\Omega)},$$

which in turn implies $J(S^+) = H(S^+)/G(S^+) \leq H(\Omega)/G(\Omega) = J(\Omega)$.

Similarly, one obtains $J(S^-) \geq J(\Omega)$, so $J(S^-) \geq J(S^+)$. However, computing $J(S^-)$ and $J(S^+)$, one obtains
$J(S^-) = r_1 x_1(\alpha - 1 - n, \alpha - 1) + r_2 x_2(\beta, \beta + p)$,
$J(S^+) = r_1 x_1(\alpha, \alpha + m) + r_2 x_2(\beta, \beta + p)$, thus $J(S^-) < J(S^+)$, but this is a contradiction.

So we conclude that at least one between cases (i) and (ii) holds. $\blacksquare$

$H(\cdot)$, $G(\cdot)$, $x_i(\cdot, \cdot)$ defined as in (Ross and Tsang '89).

Proof of Proposition 3

We prove (i); for (ii), similar arguments can be used. The two sets S^+ and S^- in Proposition 11 must be rectangles with the same height p.

The only value of p for which S^- is IR_Ω is $p = I_2^\Omega(\alpha - 1)$. The maximum possible value of p for which S^+ is IA_Ω is $p = I_2^{\Omega_{FR}}(\alpha)$. So, if $I_2^\Omega(\alpha - 1) > I_2^{\Omega_{FR}}(\alpha)$, Proposition 1 cannot be applied.

If, instead, $I_2^\Omega(\alpha - 1) \leq I_2^{\Omega_{FR}}(\alpha)$, then it is always possible to find sets S^- and S^+ that satisfy the assumptions of Proposition 1 (e.g., $p = I_2^\Omega(\alpha - 1)$, $n = m = 0$). ∎

Proof of Proposition 4

This is equivalent to the pair of inequalities

$$l_1^{\Omega_{FR}}(\beta_i - 1) \leq \alpha_{i+1} - 1\,, \tag{2}$$

$$l_2^{\Omega_{FR}}(\alpha_{i+1} - 1) \leq \beta_i - 1\,. \tag{3}$$

Let us prove, e.g., that (2) holds. It follows by the definition of $l_2^{\Omega^*}(\alpha_i)$, the monotonicity of $l_2^{\Omega^*}(\cdot)$, and Proposition 3 (i), that

$$\beta_i - 1 = l_2^{\Omega^*}(\alpha_i) \geq l_2^{\Omega^*}(\alpha_{i+1} - 1) > l_2^{\Omega_{FR}}(\alpha_{i+1})\,. \tag{4}$$

Suppose now that the inequality $l_1^{\Omega_{FR}}(\beta_i - 1) > \alpha_{i+1} - 1$ opposite to (2) holds, and let us show that this leads to a contradiction. Indeed, since α_{i+1} is an integer, one has

$$l_1^{\Omega_{FR}}(\beta_i - 1) > \alpha_{i+1} - 1 \Leftrightarrow l_1^{\Omega_{FR}}(\beta_i - 1) \geq \alpha_{i+1}\,.$$

This, combined with the (straightforward) property $l_2^{\Omega_{FR}}(l_1^{\Omega_{FR}}(\beta_i - 1)) \geq \beta_i - 1$ and the monotonicity of $l_2^{\Omega_{FR}}(\cdot)$, implies that $l_2^{\Omega_{FR}}(\alpha_{i+1}) \geq \beta_i - 1$, but this contradicts (4). So we conclude that (2) holds. The proof of (3) is similar. ■